Conformal Geometric Algebra

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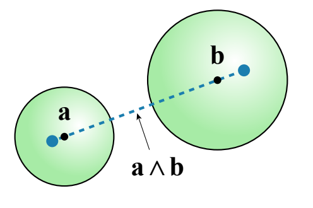
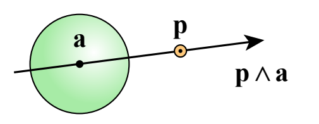
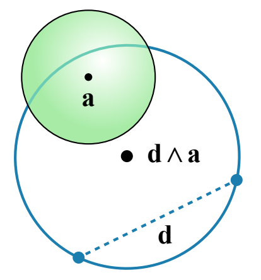
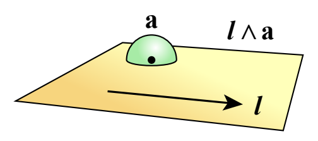
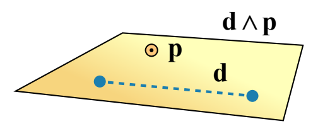
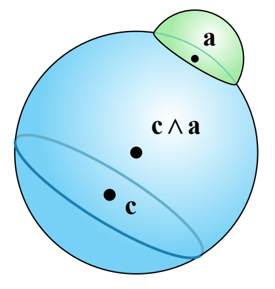
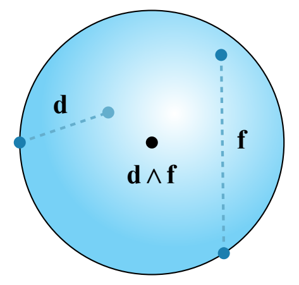
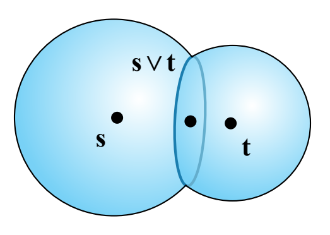
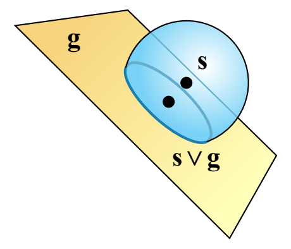
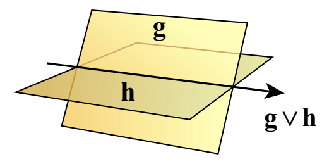
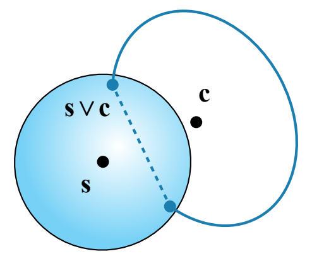
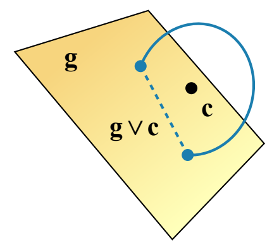
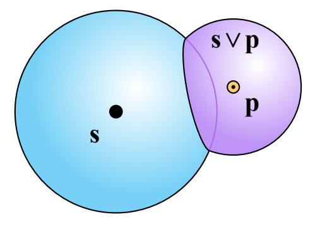
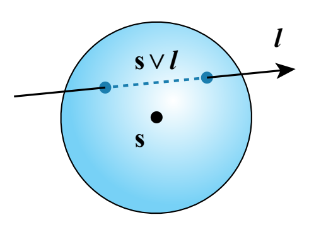
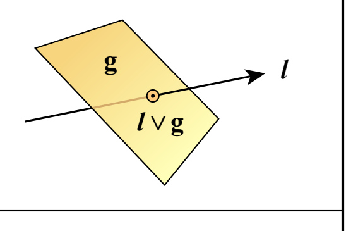
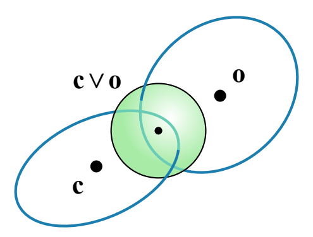
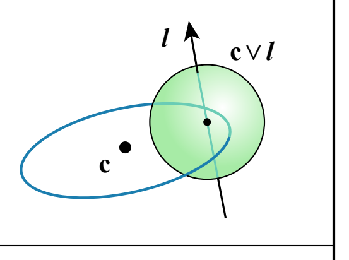
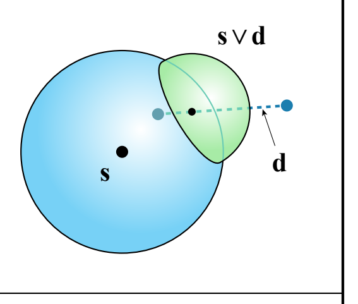
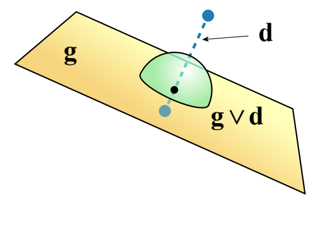
JOIN

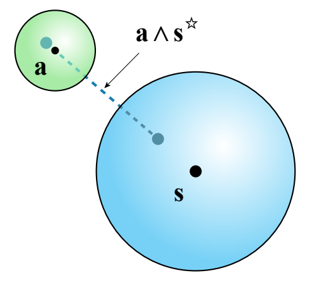
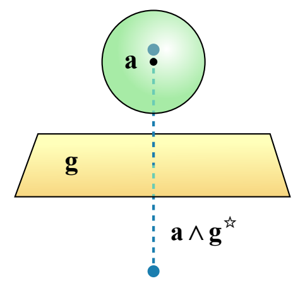
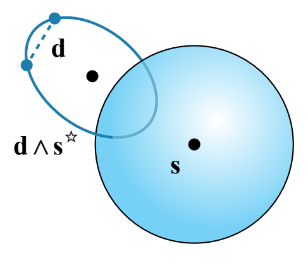
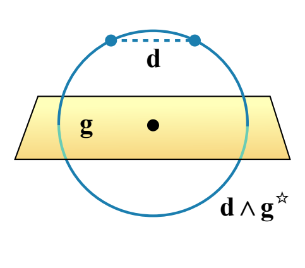
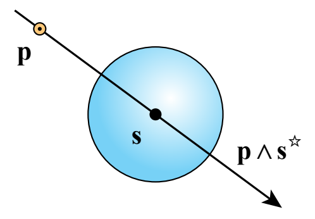
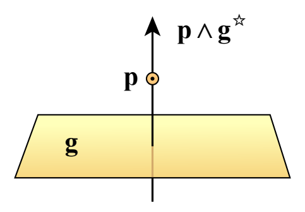
Join Operation	Illustration
Dipole containing round points **a** and **b**. $\mathbf{a}\wedge\mathbf{b} = (a_w b_x - a_x b_w)\mathbf{e}_{41} + (a_w b_y - a_y b_w)\mathbf{e}_{42} + (a_w b_z - a_z b_w)\mathbf{e}_{43} + (a_y b_z - a_z b_y)\mathbf{e}_{23} + (a_z b_x - a_x b_z)\mathbf{e}_{31} + (a_x b_y - a_y b_x)\mathbf{e}_{12} + (a_x b_u - a_u b_x)\mathbf{e}_{15} + (a_y b_u - a_u b_y)\mathbf{e}_{25} + (a_z b_u - a_u b_z)\mathbf{e}_{35} + (a_w b_u - a_u b_w)\mathbf{e}_{45}$	
Line containing flat point **p** and round point **a**. $\mathbf{p}\wedge\mathbf{a} = (p_x a_w - p_w a_x)\mathbf{e}_{415} + (p_z a_y - p_y a_z)\mathbf{e}_{235} + (p_y a_w - p_w a_y)\mathbf{e}_{425} + (p_x a_z - p_z a_x)\mathbf{e}_{315} + (p_z a_w - p_w a_z)\mathbf{e}_{435} + (p_y a_x - p_x a_y)\mathbf{e}_{125}$	
Circle containing dipole **d** and round point **a**. $\mathbf{d}\wedge\mathbf{a} = (d_{vy} a_z - d_{vz} a_y + d_{mx} a_w)\mathbf{e}_{423} + (d_{vz} a_x - d_{vx} a_z + d_{my} a_w)\mathbf{e}_{431} + (d_{vx} a_y - d_{vy} a_x + d_{mz} a_w)\mathbf{e}_{412} - (d_{mx} a_x + d_{my} a_y + d_{mz} a_z)\mathbf{e}_{321} + (d_{px} a_w - d_{pw} a_x + d_{vx} a_u)\mathbf{e}_{415} + (d_{pz} a_y - d_{py} a_z + d_{mx} a_u)\mathbf{e}_{235} + (d_{py} a_w - d_{pw} a_y + d_{vy} a_u)\mathbf{e}_{425} + (d_{px} a_z - d_{pz} a_x + d_{my} a_u)\mathbf{e}_{315} + (d_{pz} a_w - d_{pw} a_z + d_{vz} a_u)\mathbf{e}_{435} + (d_{py} a_x - d_{px} a_y + d_{mz} a_u)\mathbf{e}_{125}$	
Plane containing line ***l*** and round point **a**. $\boldsymbol{l}\wedge\mathbf{a} = (l_{vz} a_y - l_{vy} a_z - l_{mx} a_w)\mathbf{e}_{4235} + (l_{vx} a_z - l_{vz} a_x - l_{my} a_w)\mathbf{e}_{4315} + (l_{vy} a_x - l_{vx} a_y - l_{mz} a_w)\mathbf{e}_{4125} + (l_{mx} a_x + l_{my} a_y + l_{mz} a_z)\mathbf{e}_{3215}$	
Plane containing dipole **d** and flat point **p**. $\mathbf{d}\wedge\mathbf{p} = (d_{vy} p_z - d_{vz} p_y + d_{mx} p_w)\mathbf{e}_{4235} + (d_{vz} p_x - d_{vx} p_z + d_{my} p_w)\mathbf{e}_{4315} + (d_{vx} p_y - d_{vy} p_x + d_{mz} p_w)\mathbf{e}_{4125} - (d_{mx} p_x + d_{my} p_y + d_{mz} p_z)\mathbf{e}_{3215}$	
Sphere containing circle **c** and round point **a**. $\mathbf{c}\wedge\mathbf{a} = -(c_{gx} a_x + c_{gy} a_y + c_{gz} a_z + c_{gw} a_w)\mathbf{e}_{1234} + (c_{vz} a_y - c_{vy} a_z + c_{gx} a_u - c_{mx} a_w)\mathbf{e}_{4235} + (c_{vx} a_z - c_{vz} a_x + c_{gy} a_u - c_{my} a_w)\mathbf{e}_{4315} + (c_{vy} a_x - c_{vx} a_y + c_{gz} a_u - c_{mz} a_w)\mathbf{e}_{4125} + (c_{mx} a_x + c_{my} a_y + c_{mz} a_z + c_{gw} a_u)\mathbf{e}_{3215}$	
Sphere containing dipoles **d** and **f**. $\mathbf{d}\wedge\mathbf{f} = -(d_{vx} f_{mx} + d_{vy} f_{my} + d_{vz} f_{mz} + d_{mx} f_{vx} + d_{my} f_{vy} + d_{mz} f_{vz})\mathbf{e}_{1234} + (d_{vy} f_{pz} - d_{vz} f_{py} + d_{pz} f_{vy} - d_{py} f_{vz} + d_{mx} f_{pw} + d_{pw} f_{mx})\mathbf{e}_{4235} + (d_{vz} f_{px} - d_{vx} f_{pz} + d_{px} f_{vz} - d_{pz} f_{vx} + d_{my} f_{pw} + d_{pw} f_{my})\mathbf{e}_{4315} + (d_{vx} f_{py} - d_{vy} p_{px} + d_{py} f_{vx} - d_{px} f_{vy} + d_{mz} f_{pw} + d_{pw} f_{mz})\mathbf{e}_{4125} - (d_{mx} f_{px} + d_{my} f_{py} + d_{mz} f_{pz} + d_{px} f_{mx} + d_{py} f_{my} + d_{pz} f_{mz})\mathbf{e}_{3215}$	

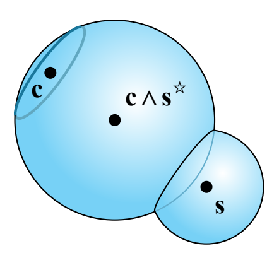
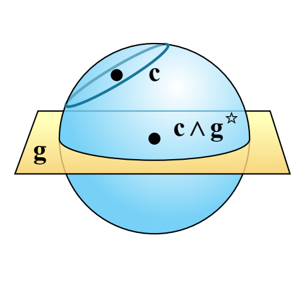
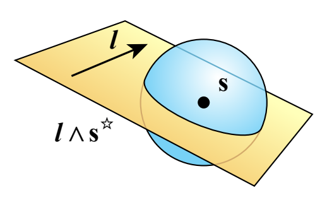
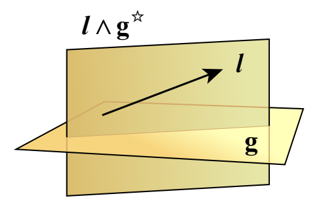
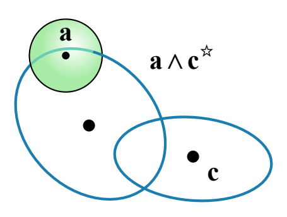
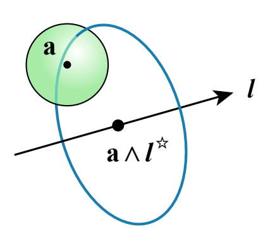
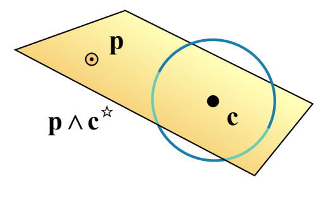
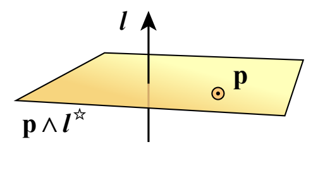
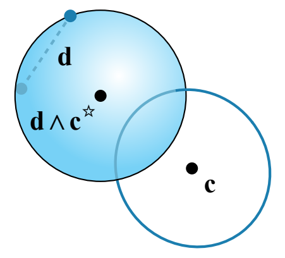
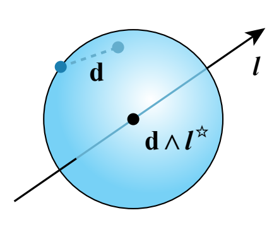
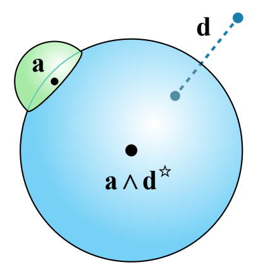
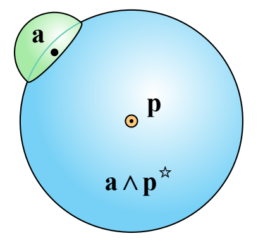
MEET

Meet Operation	Illustration
Circle where spheres **s** and **t** intersect. $\mathbf{s}\vee\mathbf{t} = (s_u t_x - s_x t_u)\mathbf{e}_{423} + (s_u t_y - s_y t_u)\mathbf{e}_{431} + (s_u t_z - s_z t_u)\mathbf{e}_{412} + (s_u t_w - s_w t_u)\mathbf{e}_{321} + (s_z t_y - s_y t_z)\mathbf{e}_{415} + (s_x t_z - s_z t_x)\mathbf{e}_{425} + (s_y t_x - s_x t_y)\mathbf{e}_{435} + (s_x t_w - s_w t_x)\mathbf{e}_{235} + (s_y t_w - s_w t_y)\mathbf{e}_{315} + (s_z t_w - s_w t_z)\mathbf{e}_{125}$	
Circle where sphere **s** and plane **g** intersect. $\mathbf{s}\vee\mathbf{g} = s_u g_x\mathbf{e}_{423} + s_u g_y\mathbf{e}_{431} + s_u g_z\mathbf{e}_{412} + s_u g_w\mathbf{e}_{321} + (s_z g_y - s_y g_z)\mathbf{e}_{415} + (s_x g_z - s_z g_x)\mathbf{e}_{425} + (s_y g_x - s_x g_y)\mathbf{e}_{435} + (s_x g_w - s_w g_x)\mathbf{e}_{235} + (s_y g_w - s_w g_y)\mathbf{e}_{315} + (s_z g_w - s_w g_z)\mathbf{e}_{125}$	
Line where planes **g** and **h** intersect. $\mathbf{g}\vee\mathbf{h} = (g_z h_y - g_y h_z)\mathbf{e}_{415} + (g_x h_w - g_w h_x)\mathbf{e}_{235} + (g_x h_z - g_z h_x)\mathbf{e}_{425} + (g_y h_w - g_w h_y)\mathbf{e}_{315} + (g_y h_x - g_x h_y)\mathbf{e}_{435} + (g_z h_w - g_w h_z)\mathbf{e}_{125}$	
Dipole where sphere **s** and circle **c** intersect. $\mathbf{s}\vee\mathbf{c} = (s_y c_{gz} - s_z c_{gy} + s_u c_{vx})\mathbf{e}_{41} + (s_w c_{gx} - s_x c_{gw} + s_u c_{mx})\mathbf{e}_{23} + (s_z c_{gx} - s_x c_{gz} + s_u c_{vy})\mathbf{e}_{42} + (s_w c_{gy} - s_y c_{gw} + s_u c_{my})\mathbf{e}_{31} + (s_x c_{gy} - s_y c_{gx} + s_u c_{vz})\mathbf{e}_{43} + (s_w c_{gz} - s_z c_{gw} + s_u c_{mz})\mathbf{e}_{12} + (s_z c_{my} - s_y c_{mz} + s_w c_{vx})\mathbf{e}_{15} + (s_x c_{mz} - s_z c_{mx} + s_w c_{vy})\mathbf{e}_{25} + (s_y c_{mx} - s_x c_{my} + s_w c_{vz})\mathbf{e}_{35} - (s_x c_{vx} + s_y c_{vy} + s_z c_{vz})\mathbf{e}_{45}$	
Dipole where plane **g** and circle **c** intersect. $\mathbf{g}\vee\mathbf{c} = (g_y c_{gz} - g_z c_{gy})\mathbf{e}_{41} + (g_w c_{gx} - g_x c_{gw})\mathbf{e}_{23} + (g_z c_{gx} - g_x c_{gz})\mathbf{e}_{42} + (g_w c_{gy} - g_y c_{gw})\mathbf{e}_{31} + (g_x c_{gy} - g_y c_{gx})\mathbf{e}_{43} + (g_w c_{gz} - g_z c_{gw})\mathbf{e}_{12} + (g_z c_{my} - g_y c_{mz} + g_w c_{vx})\mathbf{e}_{15} + (g_x c_{mz} - g_z c_{mx} + g_w c_{vy})\mathbf{e}_{25} + (g_y c_{mx} - g_x c_{my} + g_w c_{vz})\mathbf{e}_{35} - (g_x c_{vx} + g_y c_{vy} + g_z c_{vz})\mathbf{e}_{45}$	
Round point centered at flat point **p** and contained by sphere **s**. $\mathbf{s}\vee\mathbf{p} = s_u p_x\mathbf{e}_1 + s_u p_y\mathbf{e}_2 + s_u p_z\mathbf{e}_3 + s_u p_w\mathbf{e}_4 - (s_x p_x + s_y p_y + s_z p_z + s_w p_w)\mathbf{e}_5$	

Meet Operation	Illustration
Dipole where sphere **s** and line ***l*** intersect. $\mathbf{s}\vee\boldsymbol{l} = s_u l_{vx}\mathbf{e}_{41} + s_u l_{vy}\mathbf{e}_{42} + s_u l_{vz}\mathbf{e}_{43} + s_u l_{mx}\mathbf{e}_{23} + s_u l_{my}\mathbf{e}_{31} + s_u l_{mz}\mathbf{e}_{12} + (s_z l_{my} - s_y l_{mz} + s_w l_{vx})\mathbf{e}_{15} + (s_x l_{mz} - s_z l_{mx} + s_w l_{vy})\mathbf{e}_{25} + (s_y l_{mx} - s_x l_{my} + s_w l_{vz})\mathbf{e}_{35} - (s_x l_{vx} + s_y l_{vy} + s_z l_{vz})\mathbf{e}_{45}$	
Flat point where plane **g** and line ***l*** intersect. $\mathbf{g}\vee\boldsymbol{l} = (g_z l_{my} - g_y l_{mz} + g_w l_{vx})\mathbf{e}_{15} + (g_x l_{mz} - g_z l_{mx} + g_w l_{vy})\mathbf{e}_{25} + (g_y l_{mx} - g_x l_{my} + g_w l_{vz})\mathbf{e}_{35} - (g_x l_{vx} + g_y l_{vy} + g_z l_{vz})\mathbf{e}_{45}$	
Round point contained by circles **c** and **o**. $\mathbf{c}\vee\mathbf{o} = (c_{gz} o_{my} - c_{gy} o_{mz} + c_{my} o_{gz} - c_{mz} o_{gy} + c_{vx} o_{gw} + c_{gw} o_{vx})\mathbf{e}_1 + (c_{gx} o_{mz} - c_{gz} o_{mx} + c_{mz} o_{gx} - c_{mx} o_{gz} + c_{vy} o_{gw} + c_{gw} o_{vy})\mathbf{e}_2 + (c_{gy} o_{mx} - c_{gx} o_{my} + c_{mx} o_{gy} - c_{my} o_{gx} + c_{vz} o_{gw} + c_{gw} o_{vz})\mathbf{e}_3 - (c_{gx} o_{vx} + c_{gy} o_{vy} + c_{gz} o_{vz} + c_{vx} o_{gx} + c_{vy} o_{gy} + c_{vz} o_{gz})\mathbf{e}_4 - (c_{mx} o_{vx} + c_{my} o_{vy} + c_{mz} o_{vz} + c_{vx} o_{mx} + c_{vy} o_{my} + c_{vz} o_{mz})\mathbf{e}_5$	
Round point centered on line ***l*** and contained by circle **c**. $\mathbf{c}\vee\boldsymbol{l} = (c_{gz} l_{my} - c_{gy} l_{mz} + c_{gw} l_{vx})\mathbf{e}_1 + (c_{gx} l_{mz} - c_{gz} l_{mx} + c_{gw} l_{vy})\mathbf{e}_2 + (c_{gy} l_{mx} - c_{gx} l_{my} + c_{gw} l_{vz})\mathbf{e}_3 - (c_{gx} l_{vx} + c_{gy} l_{vy} + c_{gz} l_{vz})\mathbf{e}_4 - (c_{mx} l_{vx} + c_{my} l_{vy} + c_{mz} l_{vz} + c_{vx} l_{mx} + c_{vy} l_{my} + c_{vz} l_{mz})\mathbf{e}_5$	
Round point contained by sphere **s** and dipole **d**. $\mathbf{s}\vee\mathbf{d} = (s_y d_{mz} - s_z d_{my} - s_w d_{vx} + s_u d_{px})\mathbf{e}_1 + (s_z d_{mx} - s_x d_{mz} - s_w d_{vy} + s_u d_{py})\mathbf{e}_2 + (s_x d_{my} - s_y d_{mx} - s_w d_{vz} + s_u d_{pz})\mathbf{e}_3 + (s_x d_{vx} + s_y d_{vy} + s_z d_{vz} + s_u d_{pw})\mathbf{e}_4 - (s_x d_{px} + s_y d_{py} + s_z d_{pz} + s_w d_{pw})\mathbf{e}_5$	
Round point centered in plane **g** and contained by dipole **d**. $\mathbf{g}\vee\mathbf{d} = (g_y d_{mz} - g_z d_{my} - g_w d_{vx})\mathbf{e}_1 + (g_z d_{mx} - g_x d_{mz} - g_w d_{vy})\mathbf{e}_2 + (g_x d_{my} - g_y d_{mx} - g_w d_{vz})\mathbf{e}_3 + (g_x d_{vx} + g_y d_{vy} + g_z d_{vz})\mathbf{e}_4 - (g_x d_{px} + g_y d_{py} + g_z d_{pz} + g_w d_{pw})\mathbf{e}_5$	

EXPANSION

Expansion Operation	Illustration
Dipole containing round point **a** and orthogonal to sphere **s**. $\mathbf{a}\wedge\mathbf{s}^{\star} = (a_x s_u + a_w s_x)\mathbf{e}_{41} + (a_y s_z - a_z s_y)\mathbf{e}_{23} + (a_y s_u + a_w s_y)\mathbf{e}_{42} + (a_z s_x - a_x s_z)\mathbf{e}_{31} + (a_z s_u + a_w s_z)\mathbf{e}_{43} + (a_x s_y - a_y s_x)\mathbf{e}_{12} - (a_x s_w + a_u s_x)\mathbf{e}_{15} - (a_y s_w + a_u s_y)\mathbf{e}_{25} - (a_z s_w + a_u s_z)\mathbf{e}_{35} + (a_u s_u - a_w s_w)\mathbf{e}_{45}$	
Dipole containing round point **a** and orthogonal to plane **g**. $\mathbf{a}\wedge\mathbf{g}^{\star} = a_w g_x\mathbf{e}_{41} + (a_y g_z - a_z g_y)\mathbf{e}_{23} + a_w g_y\mathbf{e}_{42} + (a_z g_x - a_x g_z)\mathbf{e}_{31} + a_w g_z\mathbf{e}_{43} + (a_x g_y - a_y g_x)\mathbf{e}_{12} - (a_x g_w + a_u g_x)\mathbf{e}_{15} - (a_y g_w + a_u g_y)\mathbf{e}_{25} - (a_z g_w + a_u g_z)\mathbf{e}_{35} - a_w g_w\mathbf{e}_{45}$	
Circle containing dipole **d** and orthogonal to sphere **s**. $\mathbf{d}\wedge\mathbf{s}^{\star} = (d_{vy} s_z - d_{vz} s_y - d_{mx} s_u)\mathbf{e}_{423} + (d_{vz} s_x - d_{vx} s_z - d_{my} s_u)\mathbf{e}_{431} + (d_{vx} s_y - d_{vy} s_x - d_{mz} s_u)\mathbf{e}_{412} - (d_{mx} s_x + d_{my} s_y + d_{mz} s_z)\mathbf{e}_{321} - (d_{vx} s_w + d_{pw} s_x + d_{px} s_u)\mathbf{e}_{415} + (d_{pz} s_y - d_{py} s_z - d_{mx} s_w)\mathbf{e}_{235} - (d_{vy} s_w + d_{pw} s_y + d_{py} s_u)\mathbf{e}_{425} + (d_{px} s_z - d_{pz} s_x - d_{my} s_w)\mathbf{e}_{315} - (d_{vz} s_w + d_{pw} s_z + d_{pz} s_u)\mathbf{e}_{435} + (d_{py} s_x - d_{px} s_y - d_{mz} s_w)\mathbf{e}_{125}$	
Circle containing dipole **d** and orthogonal to plane **g**. $\mathbf{d}\wedge\mathbf{g}^{\star} = (d_{vy} g_z - d_{vz} g_y)\mathbf{e}_{423} + (d_{vz} g_x - d_{vx} g_z)\mathbf{e}_{431} + (d_{vx} g_y - d_{vy} g_x)\mathbf{e}_{412} - (d_{mx} g_x + d_{my} g_y + d_{mz} g_z)\mathbf{e}_{321} - (d_{vx} g_w + d_{pw} g_x)\mathbf{e}_{415} + (d_{pz} g_y - d_{py} g_z - d_{mx} g_w)\mathbf{e}_{235} - (d_{vy} g_w + d_{pw} g_y)\mathbf{e}_{425} + (d_{px} g_z - d_{pz} g_x - d_{my} g_w)\mathbf{e}_{315} - (d_{vz} g_w + d_{pw} g_z)\mathbf{e}_{435} + (d_{py} g_x - d_{px} g_y - d_{mz} g_w)\mathbf{e}_{125}$	
Line containing flat point **p** and orthogonal to sphere **s**. $\mathbf{p}\wedge\mathbf{s}^{\star} = -(p_w s_x + p_x s_u)\mathbf{e}_{415} + (p_z s_y - p_y s_z)\mathbf{e}_{235} - (p_w s_y + p_y s_u)\mathbf{e}_{425} + (p_x s_z - p_z s_x)\mathbf{e}_{315} - (p_w s_z + p_z s_u)\mathbf{e}_{435} + (p_y s_x - p_x s_y)\mathbf{e}_{125}$	
Line containing flat point **p** and orthogonal to plane **g**. $\mathbf{p}\wedge\mathbf{g}^{\star} = -p_w g_x\mathbf{e}_{415} + (p_z g_y - p_y g_z)\mathbf{e}_{235} - p_w g_y\mathbf{e}_{425} + (p_x g_z - p_z g_x)\mathbf{e}_{315} - p_w g_z\mathbf{e}_{435} + (p_y g_x - p_x g_y)\mathbf{e}_{125}$	

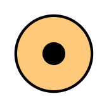
Expansion Operation	Illustration
Sphere containing circle **c** and orthogonal to sphere **s**. $\mathbf{c}\wedge\mathbf{s}^{\star} = (c_{gw} s_u - c_{gx} s_x - c_{gy} s_y - c_{gz} s_z)\mathbf{e}_{1234} + (c_{vz} s_y - c_{vy} s_z + c_{mx} s_u - c_{gx} s_w)\mathbf{e}_{4235} + (c_{vx} s_z - c_{vz} s_x + c_{my} s_u - c_{gy} s_w)\mathbf{e}_{4315} + (c_{vy} s_x - c_{vx} s_y + c_{mz} s_u - c_{gz} s_w)\mathbf{e}_{4125} + (c_{mx} s_x + c_{my} s_y + c_{mz} s_z - c_{gw} s_w)\mathbf{e}_{3215}$	
Sphere containing circle **c** and orthogonal to plane **g**. $\mathbf{c}\wedge\mathbf{g}^{\star} = -(c_{gx} g_x + c_{gy} g_y + c_{gz} g_z)\mathbf{e}_{1234} + (c_{vz} g_y - c_{vy} g_z - c_{gx} g_w)\mathbf{e}_{4235} + (c_{vx} g_z - c_{vz} g_x - c_{gy} g_w)\mathbf{e}_{4315} + (c_{vy} g_x - c_{vx} g_y - c_{gz} g_w)\mathbf{e}_{4125} + (c_{mx} g_x + c_{my} g_y + c_{mz} g_z - c_{gw} g_w)\mathbf{e}_{3215}$	
Plane containing line ***l*** and orthogonal to sphere **s**. $\boldsymbol{l}\wedge\mathbf{s}^{\star} = (l_{vz} s_y - l_{vy} s_z + l_{mx} s_u)\mathbf{e}_{4235} + (l_{vx} s_z - l_{vz} s_x + l_{my} s_u)\mathbf{e}_{4315} + (l_{vy} s_x - l_{vx} s_y + l_{mz} s_u)\mathbf{e}_{4125} + (l_{mx} s_x + l_{my} s_y + l_{mz} s_z)\mathbf{e}_{3215}$	
Plane containing line ***l*** and orthogonal to plane **g**. $\boldsymbol{l}\wedge\mathbf{g}^{\star} = (l_{vz} g_y - l_{vy} g_z)\mathbf{e}_{4235} + (l_{vx} g_z - l_{vz} g_x)\mathbf{e}_{4315} + (l_{vy} g_x - l_{vx} g_y)\mathbf{e}_{4125} + (l_{mx} g_x + l_{my} g_y + l_{mz} g_z)\mathbf{e}_{3215}$	
Circle containing round point **a** and orthogonal to circle **c**. $\mathbf{a}\wedge\mathbf{c}^{\star} = (a_y c_{gz} - a_z c_{gy} - a_w c_{vx})\mathbf{e}_{423} + (a_z c_{gx} - a_x c_{gz} - a_w c_{vy})\mathbf{e}_{431} + (a_x c_{gy} - a_y c_{gx} - a_w c_{vz})\mathbf{e}_{412} + (a_x c_{vx} + a_y c_{vy} + a_z c_{vz})\mathbf{e}_{321} - (a_x c_{gw} + a_w c_{mx} + a_u c_{gx})\mathbf{e}_{415} + (a_z c_{my} - a_y c_{mz} - a_u c_{vx})\mathbf{e}_{235} - (a_y c_{gw} + a_w c_{my} + a_u c_{gy})\mathbf{e}_{425} + (a_x c_{mz} - a_z c_{mx} - a_u c_{vy})\mathbf{e}_{315} - (a_z c_{gw} + a_w c_{mz} + a_u c_{gz})\mathbf{e}_{435} + (a_y c_{mx} - a_x c_{my} - a_u c_{vz})\mathbf{e}_{125}$	
Circle containing round point **a** and orthogonal to line ***l***. $\mathbf{a}\wedge\boldsymbol{l}^{\star} = -a_w l_{vx}\mathbf{e}_{423} - a_w l_{vy}\mathbf{e}_{431} - a_w l_{vz}\mathbf{e}_{412} + (a_x l_{vx} + a_y l_{vy} + a_z l_{vz})\mathbf{e}_{321} - a_w l_{mx}\mathbf{e}_{415} + (a_z l_{my} - a_y l_{mz} - a_u l_{vx})\mathbf{e}_{235} - a_w l_{my}\mathbf{e}_{425} + (a_x l_{mz} - a_z l_{mx} - a_u l_{vy})\mathbf{e}_{315} - a_w l_{mz}\mathbf{e}_{435} + (a_y l_{mx} - a_x l_{my} - a_u l_{vz})\mathbf{e}_{125}$	
Plane containing flat point **p** and orthogonal to circle **c**. $\mathbf{p}\wedge\mathbf{c}^{\star} = (p_y c_{gz} - p_z c_{gy} - p_w c_{vx})\mathbf{e}_{4235} + (p_z c_{gx} - p_x c_{gz} - p_w c_{vy})\mathbf{e}_{4315} + (p_x c_{gy} - p_y c_{gx} - p_w c_{vz})\mathbf{e}_{4125} + (p_x c_{vx} + p_y c_{vy} + p_z c_{vz})\mathbf{e}_{3215}$	
Plane containing flat point **p** and orthogonal to line ***l***. $\mathbf{p}\wedge\boldsymbol{l}^{\star} = -p_w l_{vx}\mathbf{e}_{4235} - p_w l_{vy}\mathbf{e}_{4315} - p_w l_{vz}\mathbf{e}_{4125} + (p_x l_{vx} + p_y l_{vy} + p_z l_{vz})\mathbf{e}_{3215}$	
Sphere containing dipole **d** and orthogonal to circle **c**. $\mathbf{d}\wedge\mathbf{c}^{\star} = (d_{vx} c_{vx} + d_{vy} c_{vy} + d_{vz} c_{vz} + d_{mx} c_{gx} + d_{my} c_{gy} + d_{mz} c_{gz})\mathbf{e}_{1234} + (d_{vz} c_{my} - d_{vy} c_{mz} - d_{pw} c_{vx} + d_{py} c_{gz} - d_{pz} c_{gy} + d_{mx} c_{gw})\mathbf{e}_{4235} + (d_{vx} c_{mz} - d_{vz} c_{mx} - d_{pw} c_{vy} + d_{pz} c_{gx} - d_{px} c_{gz} + d_{my} c_{gw})\mathbf{e}_{4315} + (d_{vy} c_{mx} - d_{vx} c_{my} - d_{pw} c_{vz} + d_{px} c_{gy} - d_{py} c_{gx} + d_{mz} c_{gw})\mathbf{e}_{4125} + (d_{px} c_{vx} + d_{py} c_{vy} + d_{pz} c_{vz} + d_{mx} c_{mx} + d_{my} c_{my} + d_{mz} c_{mz})\mathbf{e}_{3215}$	
Sphere containing dipole **d** and orthogonal to line ***l***. $\mathbf{d}\wedge\boldsymbol{l}^{\star} = (d_{vx} l_{vx} + d_{vy} l_{vy} + d_{vz} l_{vz})\mathbf{e}_{1234} + (d_{vz} l_{my} - d_{vy} l_{mz} - d_{pw} l_{vx})\mathbf{e}_{4235} + (d_{vx} l_{mz} - d_{vz} l_{mx} - d_{pw} l_{vy})\mathbf{e}_{4315} + (d_{vy} l_{mx} - d_{vx} l_{my} - d_{pw} l_{vz})\mathbf{e}_{4125} + (d_{px} l_{vx} + d_{py} l_{vy} + d_{pz} l_{vz} + d_{mx} l_{mx} + d_{my} l_{my} + d_{mz} l_{mz})\mathbf{e}_{3215}$	
Sphere containing round point **a** and orthogonal to dipole **d**. $\mathbf{a}\wedge\mathbf{d}^{\star} = (a_x d_{vx} + a_y d_{vy} + a_z d_{vz} - a_w d_{pw})\mathbf{e}_{1234} + (a_z d_{my} - a_y d_{mz} + a_w d_{px} - a_u d_{vx})\mathbf{e}_{4235} + (a_x d_{mz} - a_z d_{mx} + a_w d_{py} - a_u d_{vy})\mathbf{e}_{4315} + (a_y d_{mx} - a_x d_{my} + a_w d_{pz} - a_u d_{vz})\mathbf{e}_{4125} + (a_u d_{pw} - a_x d_{px} - a_y d_{py} - a_z d_{pz})\mathbf{e}_{3215}$	
Sphere containing round point **a** and centered at flat point **p**. $\mathbf{a}\wedge\mathbf{p}^{\star} = -a_w p_w\mathbf{e}_{1234} + a_w p_x\mathbf{e}_{4235} + a_w p_y\mathbf{e}_{4315} + a_w p_z\mathbf{e}_{4125} + (a_u p_w - a_x p_x - a_y p_y - a_z p_z)\mathbf{e}_{3215}$	

FLATS

Flat Point **p**

(Bivector)

0D



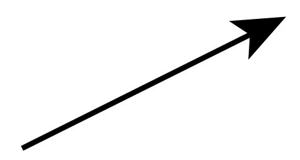
$$\mathbf{p} = p_x\mathbf{e}_{15} + p_y\mathbf{e}_{25} + p_z\mathbf{e}_{35} + p_w\mathbf{e}_{45}$$

Dual	$\mathbf{p}^{\bigstar} = p_w\mathbf{e}_{321} - p_x\mathbf{e}_{235} - p_y\mathbf{e}_{315} - p_z\mathbf{e}_{125}$
Attitude	$\operatorname{att}(\mathbf{p}) = \mathbf{p}\vee\mathbf{e}_{3215} = p_w\mathbf{e}_5$
Flat Bulk	$\mathbf{p}_{\blacksquare} = p_x\mathbf{e}_{15} + p_y\mathbf{e}_{25} + p_z\mathbf{e}_{35}$
Flat Weight	$\mathbf{p}_{\square} = p_w\mathbf{e}_{45}$
Position Norm	$\dfrac{\lVert\mathbf{p}\rVert_{\blacksquare}}{\lVert\mathbf{p}\rVert_{\square}} = \sqrt{\dfrac{p_x^2 + p_y^2 + p_z^2}{p_w^2}}$

Flat Line ***l***

(Trivector)

1D



$$\boldsymbol{l} = l_{vx}\mathbf{e}_{415} + l_{vy}\mathbf{e}_{425} + l_{vz}\mathbf{e}_{435} + l_{mx}\mathbf{e}_{235} + l_{my}\mathbf{e}_{315} + l_{mz}\mathbf{e}_{125}$$

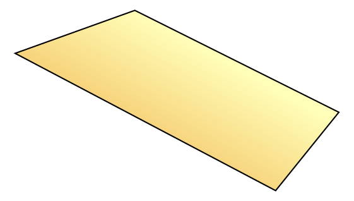
$$l_{vx} l_{mx} + l_{vy} l_{my} + l_{vz} l_{mz} = 0$$

Dual	$\boldsymbol{l}^{\bigstar} = l_{vx}\mathbf{e}_{23} + l_{vy}\mathbf{e}_{31} + l_{vz}\mathbf{e}_{12} + l_{mx}\mathbf{e}_{15} + l_{my}\mathbf{e}_{25} + l_{mz}\mathbf{e}_{35}$
Attitude	$\operatorname{att}(\boldsymbol{l}) = \boldsymbol{l}\vee\mathbf{e}_{3215} = l_{vx}\mathbf{e}_{15} + l_{vy}\mathbf{e}_{25} + l_{vz}\mathbf{e}_{35}$
Flat Bulk	$\boldsymbol{l}_{\blacksquare} = l_{mx}\mathbf{e}_{235} + l_{my}\mathbf{e}_{315} + l_{mz}\mathbf{e}_{125}$ (moment)
Flat Weight	$\boldsymbol{l}_{\square} = l_{vx}\mathbf{e}_{415} + l_{vy}\mathbf{e}_{425} + l_{vz}\mathbf{e}_{435}$ (direction)
Position Norm	$\dfrac{\lVert\boldsymbol{l}\rVert_{\blacksquare}}{\lVert\boldsymbol{l}\rVert_{\square}} = \sqrt{\dfrac{l_{mx}^2 + l_{my}^2 + l_{mz}^2}{l_{vx}^2 + l_{vy}^2 + l_{vz}^2}}$

Flat Plane **g**

(Quadrivector)

2D



$$\mathbf{g} = g_x\mathbf{e}_{4235} + g_y\mathbf{e}_{4315} + g_z\mathbf{e}_{4125} + g_w\mathbf{e}_{3215}$$

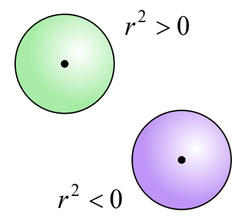
Dual	$\mathbf{g}^{\bigstar} = -g_x\mathbf{e}_1 - g_y\mathbf{e}_2 - g_z\mathbf{e}_3 + g_w\mathbf{e}_5$
Attitude	$\operatorname{att}(\mathbf{g}) = \mathbf{g}\vee\mathbf{e}_{3215} = g_x\mathbf{e}_{235} + g_y\mathbf{e}_{315} + g_z\mathbf{e}_{125}$
Flat Bulk	$\mathbf{g}_{\blacksquare} = g_w\mathbf{e}_{3215}$ (position)
Flat Weight	$\mathbf{g}_{\square} = g_x\mathbf{e}_{4235} + g_y\mathbf{e}_{4315} + g_z\mathbf{e}_{4125}$ (normal)
Position Norm	$\dfrac{\lVert\mathbf{g}\rVert_{\blacksquare}}{\lVert\mathbf{g}\rVert_{\square}} = \sqrt{\dfrac{g_w^2}{g_x^2 + g_y^2 + g_z^2}}$

ROUNDS

Round Point **a**

(Vector)

0D



$$\mathbf{a} = p_x\mathbf{e}_1 + p_y\mathbf{e}_2 + p_z\mathbf{e}_3 + \mathbf{e}_4 + \frac{p^2 + r^2}{2}\mathbf{e}_5$$

p = position

r = radius

$$\mathbf{a} = \underbrace{a_x\mathbf{e}_1 + a_y\mathbf{e}_2 + a_z\mathbf{e}_3 + a_w\mathbf{e}_4}_{\text{Carrier Point}} + \underbrace{a_u\mathbf{e}_5}_{\text{Infinity}}$$

(when $a_x = a_y = a_z = a_w = 0$)

Attitude

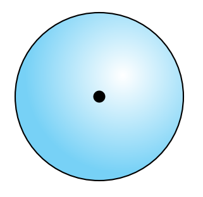
$\operatorname{att}(\mathbf{a}) = \mathbf{a}\vee\mathbf{e}_{3215} = a_w\mathbf{1}$

Center	$\operatorname{cen}(\mathbf{a}) = \operatorname{ccr}(\mathbf{a})\vee\mathbf{a} = a_x a_w\mathbf{e}_1 + a_y a_w\mathbf{e}_2 + a_z a_w\mathbf{e}_3 + a_w^2\mathbf{e}_4 + a_w a_u\mathbf{e}_5$ (identity)		
Container	$\operatorname{con}(\mathbf{a}) = \mathbf{a}\wedge\operatorname{car}(\mathbf{a})^{\star} = -a_w^2\mathbf{e}_{1234} + a_x a_w\mathbf{e}_{4235} + a_y a_w\mathbf{e}_{4315} + a_z a_w\mathbf{e}_{4125} + (a_w a_u - a_x^2 - a_y^2 - a_z^2)\mathbf{e}_{3215}$		
Dual	$\mathbf{a}^{\bigstar} = -a_w\mathbf{e}_{1234} + a_x\mathbf{e}_{4235} + a_y\mathbf{e}_{4315} + a_z\mathbf{e}_{4125} - a_u\mathbf{e}_{3215}$		
Carrier	$\operatorname{car}(\mathbf{a}) = \mathbf{a}\wedge\mathbf{e}_5 = a_x\mathbf{e}_{15} + a_y\mathbf{e}_{25} + a_z\mathbf{e}_{35} + a_w\mathbf{e}_{45}$ (flat point)		
Cocarrier	$\operatorname{ccr}(\mathbf{a}) = \mathbf{a}^{\star}\wedge\mathbf{e}_5 = a_w\mathbb{1}$ (volume)		
Round Bulk	$\mathbf{a}_{\bullet} = a_x\mathbf{e}_1 + a_y\mathbf{e}_2 + a_z\mathbf{e}_3$	Center Norm	$\dfrac{\lVert\mathbf{a}\rVert_{\odot}}{\lVert\mathbf{a}\rVert_{\circ}} = \sqrt{\dfrac{a_x^2 + a_y^2 + a_z^2}{a_w^2}}$
Round Weight	$\mathbf{a}_{\circ} = a_w\mathbf{e}_4$		
Flat Bulk	$\mathbf{a}_{\blacksquare} = a_u\mathbf{e}_5$	Radius Norm	$\dfrac{\lVert\mathbf{a}\rVert_{\odot}}{\lVert\mathbf{a}\rVert_{\circ}} = \sqrt{\dfrac{2a_w a_u - a_x^2 - a_y^2 - a_z^2}{a_w^2}}$
Flat Weight	$\mathbf{a}_{\square} = 0$		

Sphere **s**

(Quadrivector)

3D



$$\mathbf{s} = p_x\mathbf{e}_{4235} + p_y\mathbf{e}_{4315} + p_z\mathbf{e}_{4125} - \mathbf{e}_{1234} - \frac{p^2 - r^2}{2}\mathbf{e}_{3215}$$

p = center

r = radius

$$\mathbf{s} = \underbrace{s_u\mathbf{e}_{1234}}_{\text{Carrier Space}} + \underbrace{s_x\mathbf{e}_{4235} + s_y\mathbf{e}_{4315} + s_z\mathbf{e}_{4125} + s_w\mathbf{e}_{3215}}_{\text{Flat Plane}}$$

(when $s_u = 0$)

Attitude

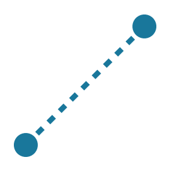
$\operatorname{att}(\mathbf{s}) = \mathbf{s}\vee\mathbf{e}_{3215} = s_u\mathbf{e}_{321} + s_x\mathbf{e}_{235} + s_y\mathbf{e}_{315} + s_z\mathbf{e}_{125}$

Center	$\operatorname{cen}(\mathbf{s}) = \operatorname{ccr}(\mathbf{s})\vee\mathbf{s} = -s_x s_u\mathbf{e}_1 - s_y s_u\mathbf{e}_2 - s_z s_u\mathbf{e}_3 + s_u^2\mathbf{e}_4 + (s_x^2 + s_y^2 + s_z^2 - s_w s_u)\mathbf{e}_5$		
Container	$\operatorname{con}(\mathbf{s}) = \mathbf{s}\wedge\operatorname{car}(\mathbf{s})^{\star} = s_u^2\mathbf{e}_{1234} + s_x s_u\mathbf{e}_{4235} + s_y s_u\mathbf{e}_{4315} + s_z s_u\mathbf{e}_{4125} + s_w s_u\mathbf{e}_{3215}$ (identity)		
Dual	$\mathbf{s}^{\bigstar} = -s_x\mathbf{e}_1 - s_y\mathbf{e}_2 - s_z\mathbf{e}_3 + s_u\mathbf{e}_4 + s_w\mathbf{e}_5$		
Carrier	$\operatorname{car}(\mathbf{s}) = \mathbf{s}\wedge\mathbf{e}_5 = s_u\mathbb{1}$ (volume)		
Cocarrier	$\operatorname{ccr}(\mathbf{s}) = \mathbf{s}^{\star}\wedge\mathbf{e}_5 = s_x\mathbf{e}_{15} + s_y\mathbf{e}_{25} + s_z\mathbf{e}_{35} - s_u\mathbf{e}_{45}$ (flat point)		
Round Bulk	$\mathbf{s}_{\bullet} = 0$	Center Norm	$\dfrac{\lVert\mathbf{s}\rVert_{\odot}}{\lVert\mathbf{s}\rVert_{\circ}} = \sqrt{\dfrac{s_x^2 + s_y^2 + s_z^2}{s_u^2}}$
Round Weight	$\mathbf{s}_{\circ} = s_u\mathbf{e}_{1234}$		
Flat Bulk	$\mathbf{s}_{\blacksquare} = s_w\mathbf{e}_{3215}$	Radius Norm	$\dfrac{\lVert\mathbf{s}\rVert_{\odot}}{\lVert\mathbf{s}\rVert_{\circ}} = \sqrt{\dfrac{s_x^2 + s_y^2 + s_z^2 - 2s_w s_u}{s_u^2}}$
Flat Weight	$\mathbf{s}_{\square} = s_x\mathbf{e}_{4235} + s_y\mathbf{e}_{4315} + s_z\mathbf{e}_{4125}$		

Dipole **d**

(Bivector)

1D



$$\mathbf{d} = n_x\mathbf{e}_{41} + n_y\mathbf{e}_{42} + n_z\mathbf{e}_{43} + (p_y n_z - p_z n_y)\mathbf{e}_{23} + (p_z n_x - p_x n_z)\mathbf{e}_{31} + (p_x n_y - p_y n_x)\mathbf{e}_{12} + (\mathbf{p}\cdot\mathbf{n})(p_x\mathbf{e}_{15} + p_y\mathbf{e}_{25} + p_z\mathbf{e}_{35} + \mathbf{e}_{45}) - \frac{p^2 + r^2}{2}(n_x\mathbf{e}_{15} + n_y\mathbf{e}_{25} + n_z\mathbf{e}_{35})$$

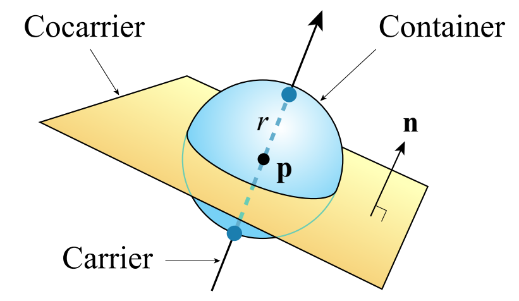
$\mathbf{p} = (p_x, p_y, p_z)$ = center

$\mathbf{n} = (n_x, n_y, n_z)$ = direction

r = radius

$$\mathbf{d} = \underbrace{\overbrace{d_{vx}\mathbf{e}_{41} + d_{vy}\mathbf{e}_{42} + d_{vz}\mathbf{e}_{43}}^{\text{Cocarrier Normal}} + d_{mx}\mathbf{e}_{23} + d_{my}\mathbf{e}_{31} + d_{mz}\mathbf{e}_{12}}_{\text{Carrier Line}} + \underbrace{d_{px}\mathbf{e}_{15} + d_{py}\mathbf{e}_{25} + d_{pz}\mathbf{e}_{35} + \overbrace{d_{pw}\mathbf{e}_{45}}^{\text{Cocarrier Position}}}_{\text{Flat Point}}$$

(when $d_{vz} = d_{vy} = d_{vz} = d_{mx} = d_{my} = d_{mz} = 0$)



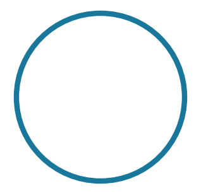
$$d_{py} d_{vz} - d_{pz} d_{vy} - d_{pw} d_{mx} = 0$$
$$d_{pz} d_{vx} - d_{px} d_{vz} - d_{pw} d_{my} = 0$$
$$d_{px} d_{vy} - d_{py} d_{vx} - d_{pw} d_{mz} = 0$$
$$d_{px} d_{mx} + d_{py} d_{my} + d_{pz} d_{mz} = 0$$
$$d_{vx} d_{mx} + d_{vy} d_{my} + d_{vz} d_{mz} = 0$$

Center	$\operatorname{cen}(\mathbf{d}) = \operatorname{ccr}(\mathbf{d})\vee\mathbf{d} = (d_{vy} d_{mz} - d_{vz} d_{my} + d_{vx} d_{pw})\mathbf{e}_1 + (d_{vz} d_{mx} - d_{vx} d_{mz} + d_{vy} d_{pw})\mathbf{e}_2 + (d_{vx} d_{my} - d_{vy} d_{mx} + d_{vz} d_{pw})\mathbf{e}_3 + (d_{vx}^2 + d_{vy}^2 + d_{vz}^2)\mathbf{e}_4 + (d_{pw}^2 - d_{vx} d_{px} - d_{vy} d_{py} - d_{vz} d_{pz})\mathbf{e}_5$				
Dual	$\mathbf{d}^{\bigstar} = -d_{vx}\mathbf{e}_{423} - d_{vy}\mathbf{e}_{431} - d_{vz}\mathbf{e}_{412} + d_{pw}\mathbf{e}_{321} - d_{mx}\mathbf{e}_{415} - d_{my}\mathbf{e}_{425} - d_{mz}\mathbf{e}_{435} - d_{px}\mathbf{e}_{235} - d_{py}\mathbf{e}_{315} - d_{pz}\mathbf{e}_{125}$			Container	$\operatorname{con}(\mathbf{d}) = \mathbf{d}\wedge\operatorname{car}(\mathbf{d})^{\star} = (d_{vx}^2 + d_{vy}^2 + d_{vz}^2)\mathbf{e}_{1234} + (d_{vz} d_{my} - d_{vy} d_{mz} - d_{vx} d_{pw})\mathbf{e}_{4235} + (d_{vx} d_{mz} - d_{vz} d_{mx} - d_{vy} d_{pw})\mathbf{e}_{4315} + (d_{vy} d_{mx} - d_{vx} d_{my} - d_{vz} d_{pw})\mathbf{e}_{4125} + (d_{mx}^2 + d_{my}^2 + d_{mz}^2 + d_{vx} d_{px} + d_{vy} d_{py} + d_{vz} d_{pz})\mathbf{e}_{3215}$
Carrier	$\operatorname{car}(\mathbf{d}) = \mathbf{d}\wedge\mathbf{e}_5 = d_{vx}\mathbf{e}_{415} + d_{vy}\mathbf{e}_{425} + d_{vz}\mathbf{e}_{435} + d_{mx}\mathbf{e}_{235} + d_{my}\mathbf{e}_{315} + d_{mz}\mathbf{e}_{125}$ (flat line)				
Cocarrier	$\operatorname{ccr}(\mathbf{d}) = \mathbf{d}^{\star}\wedge\mathbf{e}_5 = d_{vx}\mathbf{e}_{4235} + d_{vy}\mathbf{e}_{4315} + d_{vz}\mathbf{e}_{4125} - d_{pw}\mathbf{e}_{3215}$ (flat plane)				
Round Bulk	$\mathbf{d}_{\bullet} = d_{mx}\mathbf{e}_{23} + d_{my}\mathbf{e}_{31} + d_{mz}\mathbf{e}_{12}$	Attitude	$\operatorname{att}(\mathbf{d}) = \mathbf{d}\vee\mathbf{e}_{3215} = d_{vx}\mathbf{e}_1 + d_{vy}\mathbf{e}_2 + d_{vz}\mathbf{e}_3 + d_{pw}\mathbf{e}_5$		
Round Weight	$\mathbf{d}_{\circ} = d_{vx}\mathbf{e}_{41} + d_{vy}\mathbf{e}_{42} + d_{vz}\mathbf{e}_{43}$				
Flat Bulk	$\mathbf{d}_{\blacksquare} = d_{px}\mathbf{e}_{15} + d_{py}\mathbf{e}_{25} + d_{pz}\mathbf{e}_{35}$	Center Norm	$\dfrac{\lVert\mathbf{d}\rVert_{\odot}}{\lVert\mathbf{d}\rVert_{\circ}} = \sqrt{\dfrac{d_{mx}^2 + d_{my}^2 + d_{mz}^2 + d_{pw}^2}{d_{vx}^2 + d_{vy}^2 + d_{vz}^2}}$	Radius Norm	$\dfrac{\lVert\mathbf{d}\rVert_{\odot}}{\lVert\mathbf{d}\rVert_{\circ}} = \sqrt{\dfrac{d_{pw}^2 - d_{mx}^2 - d_{my}^2 - d_{mz}^2 - 2(d_{px} d_{vx} + d_{py} d_{vy} + d_{pz} d_{vz})}{d_{vx}^2 + d_{vy}^2 + d_{vz}^2}}$
Flat Weight	$\mathbf{d}_{\square} = d_{pw}\mathbf{e}_{45}$				

Circle **c**

(Trivector)

2D



$$\mathbf{c} = n_x\mathbf{e}_{423} + n_y\mathbf{e}_{431} + n_z\mathbf{e}_{412} + (p_y n_z - p_z n_y)\mathbf{e}_{415} + (p_z n_x - p_x n_z)\mathbf{e}_{425} + (p_x n_y - p_y n_x)\mathbf{e}_{435} + (\mathbf{p}\cdot\mathbf{n})(p_x\mathbf{e}_{235} + p_y\mathbf{e}_{315} + p_z\mathbf{e}_{125} - \mathbf{e}_{321}) - \frac{p^2 - r^2}{2}(n_x\mathbf{e}_{235} + n_y\mathbf{e}_{315} + n_z\mathbf{e}_{125})$$

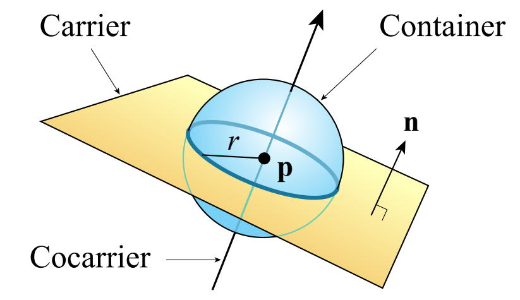
$\mathbf{p} = (p_x, p_y, p_z)$ = center

$\mathbf{n} = (n_x, n_y, n_z)$ = normal

r = radius

$$\mathbf{c} = \underbrace{\overbrace{c_{gx}\mathbf{e}_{423} + c_{gy}\mathbf{e}_{431} + c_{gz}\mathbf{e}_{412}}^{\text{Cocarrier Direction}} + c_{gw}\mathbf{e}_{321}}_{\text{Carrier Plane}} + \underbrace{\overbrace{c_{vx}\mathbf{e}_{415} + c_{vy}\mathbf{e}_{425} + c_{vz}\mathbf{e}_{435}}^{\text{Cocarrier Moment}} + c_{mx}\mathbf{e}_{235} + c_{my}\mathbf{e}_{315} + c_{mz}\mathbf{e}_{125}}_{\text{Flat Line}}$$

(when $c_{gx} = c_{gy} = c_{gz} = c_{gw} = 0$)



$$c_{gy} c_{mz} - c_{gz} c_{my} - c_{gw} c_{vx} = 0$$
$$c_{gz} c_{mx} - c_{gx} c_{mz} - c_{gw} c_{vy} = 0$$
$$c_{gx} c_{my} - c_{gy} c_{mx} - c_{gw} c_{vz} = 0$$
$$c_{vx} c_{mx} + c_{vy} c_{my} + c_{vz} c_{mz} = 0$$
$$c_{gx} c_{vx} + c_{gy} c_{vy} + c_{gz} c_{vz} = 0$$

Center	$\operatorname{cen}(\mathbf{c}) = \operatorname{ccr}(\mathbf{c})\vee\mathbf{c} = (c_{gy} c_{vz} - c_{gz} c_{vy} - c_{gx} c_{gw})\mathbf{e}_1 + (c_{gz} c_{vx} - c_{gx} c_{vz} - c_{gy} c_{gw})\mathbf{e}_2 + (c_{gx} c_{vy} - c_{gy} c_{vx} - c_{gz} c_{gw})\mathbf{e}_3 + (c_{gx}^2 + c_{gy}^2 + c_{gz}^2)\mathbf{e}_4 + (c_{vx}^2 + c_{vy}^2 + c_{vz}^2 + c_{gx} c_{mx} + c_{gy} c_{my} + c_{gz} c_{mz})\mathbf{e}_5$				
Dual	$\mathbf{c}^{\bigstar} = c_{gx}\mathbf{e}_{41} + c_{gy}\mathbf{e}_{42} + c_{gz}\mathbf{e}_{43} + c_{vx}\mathbf{e}_{23} + c_{vy}\mathbf{e}_{31} + c_{vz}\mathbf{e}_{12} + c_{mx}\mathbf{e}_{15} + c_{my}\mathbf{e}_{25} + c_{mz}\mathbf{e}_{35} - c_{gw}\mathbf{e}_{45}$			Container	$\operatorname{con}(\mathbf{c}) = \mathbf{c}\wedge\operatorname{car}(\mathbf{c})^{\star} = -(c_{gx}^2 + c_{gy}^2 + c_{gz}^2)\mathbf{e}_{1234} + (c_{gy} c_{vz} - c_{gz} c_{vy} - c_{gx} c_{gw})\mathbf{e}_{4235} + (c_{gz} c_{vx} - c_{gx} c_{vz} - c_{gy} c_{gw})\mathbf{e}_{4315} + (c_{gx} c_{vy} - c_{gy} c_{vx} - c_{gz} c_{gw})\mathbf{e}_{4125} + (c_{gx} c_{mx} + c_{gy} c_{my} + c_{gz} c_{mz} - c_{gw}^2)\mathbf{e}_{3215}$
Carrier	$\operatorname{car}(\mathbf{c}) = \mathbf{c}\wedge\mathbf{e}_5 = c_{gx}\mathbf{e}_{4235} + c_{gy}\mathbf{e}_{4315} + c_{gz}\mathbf{e}_{4125} + c_{gw}\mathbf{e}_{3215}$ (flat plane)				
Cocarrier	$\operatorname{ccr}(\mathbf{c}) = \mathbf{c}^{\star}\wedge\mathbf{e}_5 = -c_{gx}\mathbf{e}_{415} - c_{gy}\mathbf{e}_{425} - c_{gz}\mathbf{e}_{435} - c_{vx}\mathbf{e}_{235} - c_{vy}\mathbf{e}_{315} - c_{vz}\mathbf{e}_{125}$ (flat line)				
Round Bulk	$\mathbf{c}_{\bullet} = c_{gw}\mathbf{e}_{321}$	Attitude	$\operatorname{att}(\mathbf{c}) = \mathbf{c}\vee\mathbf{e}_{3215} = c_{gx}\mathbf{e}_{23} + c_{gy}\mathbf{e}_{31} + c_{gz}\mathbf{e}_{12} + c_{vx}\mathbf{e}_{15} + c_{vy}\mathbf{e}_{25} + c_{vz}\mathbf{e}_{35}$		
Round Weight	$\mathbf{c}_{\circ} = c_{gx}\mathbf{e}_{423} + c_{gy}\mathbf{e}_{431} + c_{gz}\mathbf{e}_{412}$				
Flat Bulk	$\mathbf{c}_{\blacksquare} = c_{mx}\mathbf{e}_{235} + c_{my}\mathbf{e}_{315} + c_{mz}\mathbf{e}_{125}$	Center Norm	$\dfrac{\lVert\mathbf{c}\rVert_{\odot}}{\lVert\mathbf{c}\rVert_{\circ}} = \sqrt{\dfrac{c_{gw}^2 + c_{vx}^2 + c_{vy}^2 + c_{vz}^2}{c_{gx}^2 + c_{gy}^2 + c_{gz}^2}}$	Radius Norm	$\dfrac{\lVert\mathbf{c}\rVert_{\odot}}{\lVert\mathbf{c}\rVert_{\circ}} = \sqrt{\dfrac{c_{vx}^2 + c_{vy}^2 + c_{vz}^2 - c_{gw}^2 + 2(c_{gx} c_{mx} + c_{gy} c_{my} + c_{gz} c_{mz})}{c_{gx}^2 + c_{gy}^2 + c_{gz}^2}}$
Flat Weight	$\mathbf{c}_{\square} = c_{vx}\mathbf{e}_{415} + c_{vy}\mathbf{e}_{425} + c_{vz}\mathbf{e}_{435}$				

By Eric Lengyel